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The strong ratio limit property of discrete-time Markov chains

Abstract Many of Phil Pollett's contributions to applied probability – among which the few I was involved in – concern *quasistationary distributions.* After reminiscing a little on our collaboration in this area, I will broach a related topic, the *strong ratio limit property.*

A discrete-time Markov chain on the nonnegative integers (assumed to be homogeneous, irreducible and aperiodic, but not necessarily stochastic) with matrix $P^{(n)}$ of n-step transition probabilities is said to have the strong ratio limit property (SRLP) if there exist positive constants R, $\mu(i)$ and $f(i)$ such that

$$\lim_{n\to\infty} \frac{P^{(n+m)}(i,j)}{P^{(n)}(k,l)} = R^{-m}\frac{f(i)\mu(j)}{f(k)\mu(l)}, \quad i,j,k,l \in \mathbb{N}_0,\ m \in \mathbb{Z}.$$

(Under suitable circumstances the constants $\mu(i)$ may be normalized to constitute a quasistationary distribution.) The SRLP was enunciated in the setting of recurrent Markov chains by Orey (1961) and introduced in the more general setting at hand by Pruitt (1965). Since then the problems of finding necessary and/or sufficient conditions on $P \equiv P^{(1)}$ for the corresponding Markov chain to possess the SRLP, and of identifying the constants, has received quite some attention in the literature, albeit mainly in the literature of the 20th century. The most general results to date have been obtained by Kesten (1995) and Handelman (1999).

In the talk I will give some information on the history of these problems and discuss some approaches towards their solutions.